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A Note on Goosebumps: Biological Views Versus Isoperimetric Inequalities

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Abstract. In this note, I explore various explanations of the goosebump effect. Given that this effect happens mainly on limbs, I show that it is essentially due to the protection of an animal against cold, using isoperimetric inequalities.

Résumé. Dans cette note, j'explore diverses explications de l'effet de chair de poule. Compte tenu que ce phénomène survient surtout sur les membres, je montre qu'il est essentiellement dû à la protection d'un animal contre le froid, en utilisant les inégalités isopérimétriques.

In this note, mathematics will be simple, involving only quadratic or cubic equations, but with five parameters. Its interest lies in its conceptual framework and in the fact that what we propose is counter-intuitive. Indeed, most people, and even most mathematicians, would naively think that goosebumps increase the area of the surface while keeping its volume constant. But it is not always the case, as we will see.

Goosebumps are due to the contraction of small muscles at each interior base point of a feather or a hair that pushes it outward. It produces, seen with a 20 times microscope, a perfect half-sphere on the skin. It usually happens for all birds and mammals when the body reacts to a cold environment. It also happens in order to increase, in some species of birds, the size of the body so as to impress an enemy. It might also happen when an animal is subject to fear. It is one of the main sudden changes in the appearance of an animal, that happens just in a few seconds, and therefore should deserve our attention.

The usual explanation for this fascinating phenomenon is that, for instance in reaction to cold, the animal pushes outside as far as possible the hair or feather so as to produce a thicker air mass between the feather (or hair) and the body. Obvioulsly, although human beings do not have feathers and essentially no hair, except on the head, this explanation is unsatisfactory for us, especially given that goosebumps never happen on our head, where is located our main mass of

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hair. But, of course, we could think that goosebumps in human beings would be derived from evolutionary biology, so that it would still exist although it is essentially useless.

Let's first review its biological explanation, which is the only one available. It is not completely evident that, with same hair (or feather) mass, the insulation against cold is better if made thicker. I recall here that insulation is by definition restraining the transfer of heat from one side to the other, say between the interior and the exterior of the skin of an animal. What biologists say is that the air mass increases with goosebumps at the immediate outside part of the skin (which is obviously true) so it improves the insulation. This might be true. But to prove it, one would need to test it in a laboratory where, on two milieux, one made of water, say, the other with air, both separated by an interface. If Interface 1 is a plate of say mass M, and if Interface 2 is made of the same insulating material, with the same mass M, but with the only difference that its thickness is augmented, and therefore its density is reduced, and assuming that the increase of volume for Interface 2 is entirely due to air surplus in Interface 2, one would see if Interface 2 is better at reducing the transfer of heat. I could not find an experiment of that sort in the literature. Note that, even if this experiment were conclusive, this would still not prove that the goosebumps are effective against cold, as claimed by biologists, since the air flow in the feathers is open to the outside cold air, at least when the goose flies, which is not the cases for air confined in a given material. I will come back to this question at the end of this note, and show that the explanation by biologists is right.

However, goosebumps due to our reaction to cold do not happen on the rounder part of our body. For instance, our head is essentiallly round and there is no goosebumps. This is also the case for most animals. Goosebumps do not happen either on the other round part of our body, our belly. This also applies to most animals. Of course, for the belly, this might be due to the fact that there is no feather at that place for human beings. For birds, the belly is essentially covered with fine hair.

What we see is that, in most animals, goosebumps appear essentially on the four limbs, that is to say precisely on the parts of the body that are the least round. And since I believe (this is the hypothesis of this note) that goosebumps are essentially a reaction to cold (that have served for other purposes later in the evolution), let me suggest a different explanation for goosebumps. I am not saying that this explanation excludes the explanation by biologists, I simply say that it has a significant impact.

Main Statement of this note: Under the last hypothesis, goosebumps is a phenomenon mainly, or at least partly, due to the isoperimetric inequalities in dimension 3, that is for a 3-dimensional body in a 3-dimensional environment, whose surface is therefore 2-dimensional. Actually, this statement applies to all dimensions. Goosebumps appear exactly where the body can decrease its ratio area/volume in order to reduce the loss of heat due to transfer from the inside to the outside.

Before proving this statement, let me first recall an observation about the two roundest mammals in the world: the elephant and the hippopotamus. By the isoperimetric inequality, the rounder an animal is, the less surface it has compared to its volume. Since both of these animals live in Africa (except some elephants in Asia) where they live for a few months at high temperatures, and since they must keep their temperature constant, at about 37 degrees like us, the radiation of heat outside their body is problematic. The elephant does it with its exceptionally large ears: the blood from the entire body circulates through the ears, and expels the heat (this works only when the outside temperature is lower that the temperature of the body, but this is almost always the case). For the hippopotamus, and since the water has a high degree of inertia with small temperature changes between the night and the day, this animal survives by baths every day in its closest river. In passing, this is why the hippopotamus is, with the crocodile, the most dangerous animal in Africa: one should never walk between a hippopotamus and its river.

Let me make another remark: on the parts of the body that are already round, the head and the belly, goosebumps have a counter effect: it increases the ratio area/volume. This is why there is no goosebump on heads, and this is why no animals have feather on their bellies. Actually, birds have fine hair on their belly, and this makes the effect of goosebumps (assuming that it exists on bellies) negligeable on the ratio. This is because hair is fine, and if there are thousands of minuscule bumps, this is not far from a perfect sphere. However, as we will see later on, the effect of goosebumps on the limbs, and especially on the wings, reduces the ratio area/volume inasmuch as the thickness of the wing is small, its length x is large, and the number n of feathers is limited compared to x.

Note also that in all animals endowed with limbs and a single heart (some animals have several hearts, for instance the worms), survival against the cold implies that the central part of the body is privileged. Indeed, in human beings for instance, there are valves that reduce the flow in the arteries that bring blood to the limbs. This is why, in a very cold temperature, the risk is higher for limbs. Summarizing this, in a cold environment, the limbs suffer more than the rest of the body (head and belly) for two reasons: they are further out from the heart and less important for survival (whence the existence of valves) and they are not round at all. I claim that this explains, at least partly, the goosebumps phenomenon.

In all cases, goosebumps seem to be useless in human beings. It is no longer useful for the reason given by biologists (increasing the hair fur and impressing potential enemies) and it is counter-productive for the ratio area to volume. The question is therefore: Why have goosebumps not disappeared earlier in the evolution of primates?

1. The Goosebumps Effect at the Surface of the Skin, Ignoring the Hair and Feathers If anyone has observed the wings of a goose or other poultry, without feathers, one sees that it is essentially a very flat rectangle, at least when it is extended as it is during flights. What I will now prove is that a flat rectangle is where the goosebumps are most effective to reduce the ratio area to volume, as soon as the rectangle has length much bigger than its thickness. I will take the following data: a goose wing, once all feathers or hair have been removed, is one centimeter thick, and 90 centimeters long (I do not care about its width). It has 24 feathers in a row. Note that the thickness of one centimeter is from flesh to flesh, not counting the larger thickness at the bones. This ratio 90 to 1 is even larger in bats and kites.

With a dynamical and geometric view, take two round circles in the plane, disjoint. They both realize the least ratio perimeter to area. Now bring them closer so that they have one point in common on their boundaries. Remove that point and let them be a new single body. This shape is far from being optimal, that is to say far from being a circle, and the new body will become a larger disk, as we let the area flow from one to the other. So a sequence of n disks all attached by points on their boundairies, is far from realizing the smallest ratio surface to volume, except if $n = 1$. However, this sequence is better than a flat rectangle, as long as it is quite flat, i.e. the ratio of one side to the other is sufficiently far from 1, and n is not too large. As an interesting consequence of this observation, young animals, say young dogs or young cats, or young human beings, are not as round as their parents. This is why, in a cold environment, say during the night, they must agglomerate to form an ensemble as close as possible to a round ball. In the same vein, a couple of adults, when it is freezing cold, will do the same, not to give each other their own heat (which is the same at 37 degrees) but to form a new ensemble closer to a ball, therefore reducing their total surface area with respect to the sum of their two volumes. In other words, as a funny remark, love in a cold environment is also the consequence of the isoperimetric inequality.

In order to simplify the computations, I will reduce that problem from dimension 3 to dimension 2. So let's denote by $z = 1$ the thickness of the wing, x its length and y its width. As we see on a goose's wing, the feathers are aligned in two rows. Therefore, it is enough to consider only one of these rows. So it is enough to consider a 2-dimensional rectangle R made of the (z, x) plane. That is to say, I will take a slice of the wing, keeping only the rectangle made of the thickness of the wing and its length.

Observe that goosebumps appear only in warm-blooded animals. For instance, in cold-blooded fishes, this phenomenon does not appear, which is an indication of the fact that fear or increasing its body to impress enemies are probably not the reason for the existence of goosebumps. We think that it is primarily due to a reaction to cold (but this does not exclude the use of that phenomenon for other purposes later in evolutionary biology). Moreover, it appears mainly at the regions of the body that are far from being round, that is to say the limbs.

What do goosebumps do: they replace the wing by a new body (here I only consider the body limited by flesh, not the body with feathers) with the same volume but with bumps which, examined with a microscope, are quite exactly half-balls. In order to simplify the reasoning, but without changing its rigour, assume that these half-balls on both sides (under and over) of the wing, are just a sequence of balls attached in a sequence, that is to say on a line, so that each ball is attached to the next one by just a common point on their boundaries. We must prove that under these conditions, goosebumps decrease substantially the ratio area/volume, which is the best strategy to decrease the heat flow from inside to outside and thus maintaining a constant temperature in a cold environment.

I will prove this first in dimension 2 and later on in dimension 3.

Here we compare, in a a slice view, the thickness of the wing with respect to its length in order to reduce the 3-dimensional problem to a 2-dimensional one (this is why we do not care about the width of the wing). In this respect, the 24 feathers on the wing of a goose correspond, with the goosebumps effect, to 24 disks attached together in a row, because the goosebumps effect appears over and underneath the wing, or on the edge, and these two half-spheres produce, on a very thin wing, a sequence of spheres.

In general, let's take a rectangle R in the plane whose height is 1 and length is x. Its area is x and its perimeter is $2x+2$. Now suppose that we decompose this rectangle into a sequence C of n disks, all attached in a sequence, each one being attached to the next one by a single point on their boundary, and so that the total area of these disks be the same, that is x. Its perimeter is $n(2\pi r)$. But we know that its area is:

$$Area(C) = x = n\pi r^2$$

or

$$n = \frac{x}{\pi r^2}$$

which gives $r = \sqrt{x/n\pi}$ and therefore $Perimeter(C) = n(2\pi r) = \frac{x}{\pi r^2} 2\pi r = \frac{2x}{r}$.

Hence $Perimeter(R)/Area(R) = 2 + (2/x)$ and $Perimeter(C)/Area(C) = \frac{2x/r}{x} = 2/r$. Thus, in order to compare these two ratios, let's see, in terms of x and n, where they are equal. Replacing r by its value $\sqrt{x/n\pi}$, we get

$$x^2 + x(2 - n\pi) + 1 = 0$$

so that:

$$x = \frac{n\pi - 2 \pm \sqrt{(2 - n\pi)^2 - 4}}{2}$$

and in terms of n:

$$n = \frac{2 + x + (1/x)}{\pi}.$$

This means that when n is lower than this expression in terms of x, the ratio perimeter to area for C is better, i.e lower. For instance, if n is 1, then R is replaced by a disk which realizes the best ratio. But for n larger than this expression, the rectangle R is better than C.

So now let's enter the data for a goose wing. On a slice, we get, say, 1 centimeter for the thickness and about 90 centimeters for the length, which means that $x = 90$ in this case. There are 24 goosebumps corresponding to the 24 feathers of that wing. In the last expression for n, let's replace x by 90. This gives n equal to about $92/\pi$, and so about 29.28, which is substantielly larger than 24. This means that the number of 24 feathers, and therefore 24 goosebumps, lies in the range $1 \leq n \leq 29.28$, and thus reduces the ratio, allowing the wings to better fight the cold.

But, a contrario, a similar computation for the human limbs leads to an increase of the ratio area to volume, even on our limbs, and is therefore conter-productive in reaction to cold. This is essentially due to the fact that x is much smaller, and n too lage in the case of human limbs. However, this effect is almost negligeable since human beings need hair only on the head (protection against the sun). So this seems to suggest that goosebumps are efficient for animals with wings, but not for the other ones. An explanation of this could be the fact that birds are amongst the few rare cases of life going directly back to much older times (dinosaurs) so that goosebumps might have appeared in mammals with large limbs for other reasons, or just as a passive and useless vestige of the past, like the coccyx.

I made the computations on a slice, by reduction to dimension 2. Now let's compute the real 3-dimensional problem on the wing of a goose. In general, the problem is to compare a rectangular parallelepiped P with length x, width y and thickness z, to a sequence C of n round balls of equal radius r attached to each other in a row (these are the goosebumps) that has the same volume as P, i.e. xyz. For P, its volume is xyz and its area is $2(xy+xz+yz)$. For C, its volume is still xyz and its area is $Area(C) = n(4\pi r^2)$. Thus we must compare both ratios of area to volume. Since the volumes are the same, and since we wish to see where the two quotients differ, we must compare the two numerators, leading to the equation:

$$2(xy + xz + yz) = 4n\pi r^2$$

which involves four parameters: x, y, z, n and r.

Since the volumes are equal, we get:

$$n = \frac{xyz}{\frac{4}{3}\pi r^3}$$

that gives the expression of n in terms of r.

Now, let's enter the data for a goose wing in centimeters: $x = 90$, $y = 5$, $z = 1$, and $n = 24$.

As we have seen, $nr^3 = \frac{3xyz}{4\pi} = \frac{3\times 450}{4\pi}$ which is about 107.4 with the above data. Since $n = 24$, we obtain: r^3 approximately equal to 4.475 so that r^2 is about 2.7225.

Now let's compare the areas, that is to say the isoperimetric inequalities for both P and C. For P, the area is $2(450 + 90 + 5) = 1090$ while for C it is $4 \times 24 \times \pi \times 2.7225 = 821.1$, which is quite small compared to the area of P.

This is certainly enough to suggest that goosebumps are due, at least partly, to a reduction of the area of the wing while keeping the same volume, and should be considered seriously as a reason for this phenomenon in a cold environment.

2. The Goosebumps Effect, Taking into Account the Layer of Feathers Note that all of what we have said so far applies to the goose while it is flying, and only considering its skin without feathers since we considered the wings deployed, and therefore as flat rectangles. We will now consider the goose at rest, with its feathers enveloping a round body. This will confirm the usual explanation of biologists, but this time seen in the light of the isoperimetric inequality. Assume that at rest, and forgetting the head and the legs, the goose is round. The goosebumps effect pushes out the feathers by a distance equal to the radius of the bumps, call it ϵ which is about 1 centimeter (but its exact value has no importance as we will see). Assume moreover that the air contained in the layer made between the skin and the feathers is at the same temperature as the interior of the body. Thus one may consider the body as including that air space. This means that the goosebump effect just increases the radius of the body. But since the ratio area to volume is of the order of $1/r$ for a ball, the bigger the round body is, the smaller the ratio area to volume is. However, this comparison between a big ball and a small ball is quite tricky, since we compare an area to a volume, not the same dimensions. For instance, in physics or in geometry, one cannot compare a length to a surface, and one cannot compare an energy to a mass. This is a well-known problem in science, called homogeneity. For instance, the equation $E = mc^2$ is valid since both members have the same physical units, energy, whence the necessity of multiplying the mass by a quantity that has the dimension $distance^2/time^2$. In mathematics, this question arises as well. It was solved by Descartes who introduced an artifcial unit 1 in order to compare two polynomials of different degrees. In this respect, when he considered say $x^3 - x^2 + x = 0$, therefore comparing volumes, areas and lengths, which is absurd, he defined it as $x^3 - 1 \times (x)^2 + 1 \times 1 \times x = 0$. This is the same problem here in isoperimetric inequalities. We must homogenize them, in order to compare areas to volumes for two balls of different radii. Therefore the only solution to this problem of homogeneity is to do exactly what Descartes did. Note that there is no problem at all if the volume of a body is given, and we look for the smallest area. But if we wish to compare two bodies of different volumes, we must homegenize. Therefore, in the case of the feathers of a goose at rest, we will assume that the thickness of the surface made by the feather is the same with or without goosebumps, and that this thickness is $\epsilon > 0$, very small. Doing this enables us to compare, in the isoperimetric inequality, volumes to volumes. The first volume is the area multiplied by ϵ, this is a volume, and the other one is the volume of the body. Once these considerations in mind, the answer is obvious. A round ball of larger radius r' is better in the cold than a ball of smaller radius r. Indeed, we get, for the first one the quotient of volumes:

$$4\epsilon\pi(r')^2/(4/3)\pi(r')^3$$

and for the second one the same with r' replaced by r.

Conclusion: Goosebumps have two effects that both reduce the quotient area to volume. The first one happens when the goose flies, and in this case, the increased air flow on the deployed feathers has no effect since in this position, the air flow is at the exterior temperature. But goosebumps have an effect directly on the skin of the wings, reducing the area to volume ratio. When the goose is at rest, its body is close to a round ball, and in this case, goosebumps increase the radius of the ball, whereas here the boundary of the ball is not the skin but the feathers, thus reducing the quotient of the volume near the boundary of the ball to the full volume of the ball.

It is interesting to note that goosebumps refer to gooses, as well as the French equivalent "chair de poule" that refers to the hen's skin. These animals are the best examples of the elementary theory developed in this short note.

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